

Using Z-Transform for Electric Loads Modeling in the Presence of Harmonics

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Abstract

An approach based on Z-transform is presented to obtain a single phase electric loads model in the presence of harmonics. The impedance or admittance of the load is expressed as a linear autoregressive exogenous input (ARX). The parameters of this series are identified using the least error squares (LES) algorithm. This algorithm uses the digitized samples of the load-input voltage and the resulting output current. Having identified the ARX parameters model, the impedance, or admittance of the equivalent circuit of the load can be written in the time domain. Using the well-known definition of the perunit Z, the admittance of the load can be expressed in the frequency domain. The proposed algorithm can easily be used to model the impedance or admittance of the electric distribution network on the harmonic polluted environment, as well as the transmission network, for load flow, stability analysis and harmonics filter design. The results of simulated and actual recorded data are reported in this paper.

Keywords: Z transform, electric load modeling, harmonics, LES estimation

I. INTRODUCTION

The widespread use of power electronics in transmission and distribution of electric systems as well as nonlinear loads such as electric drives, arc furnace, all kind of lighting devices produce harmonics in the power networks and an accurate model of power system components is essential. Modeling of electric loads in the presence of harmonics pays the attention of researchers and power engineering utility companies to predict the load effects during power system transients as well as steady state power flow analysis.

Many techniques are developed in the last two decades, and are grouped into two categories; the time domain approach and frequency domain approach. Some of these techniques have been applied to model the nonlinear single-phase and three phase loads.

Reference [1] presents a noninvasive method for modeling three-phase harmonic load incorporating harmonic sources for composite power system loads. The proposed technique is based on the presence of a dominant harmonic source that closes to the load. The method is shown to perform quite well in a laboratory test, but it fails when applied to modeling of an industrial load.

References [2] and [12] present a technique for modeling linear or nonlinear loads in the presence or absence of harmonics in the time-domain. The principle of energy stored in the load elements is applied. The residual currents as well as the mismatch current for different harmonic frequencies in the

voltage or current waveforms are calculated. The accuracy of the model produced in this reference is tested using the HARMFLO program that calculates the power flow at different harmonic orders.

A method for modeling distribution load impedances as unbalanced three phase impedance matrices for detailed harmonic studies is presented in Reference [3]. The linear time series is used, the parameters of this series are calculated using the least error squares algorithm and they are considered constant during the horizon time

References [4]-[6] propose different approach for modeling nonlinear dynamic loads. The load is presented by a set of differential equations that relate the active power and load voltage. The models developed are used for voltage stability, and transient studies.

Reference [7] developed a computer program that simulates the effect of harmonics introduced at distribution level by nonlinear loads. A table of harmonics (amplitude and phase) represents each load device, of the current when energized by a sinusoidal voltage. The purpose of the program is to allow load-flow studies to include effects of harmonics particularly on the customer supply voltage. The three-phase load in the program is assumed to be balanced.

The time domain controlled voltage source is presented in Reference [8] for modeling the arc furnace load. The developed model is based on a piece wise linear approximation of the $v-i$ characteristic of arc furnace load. The active power

consumed by the load is considered in the proposed load model, thus making the proposed load model depending on the operating conditions of the load.

Reference [9] describes an improved model of rotating loads in complex load areas, for harmonic penetration analyses applied to distribution systems. The analysis is performed in the frequency domain. The proposed model can be used for simulating large complex load areas within harmonic studies for which the injection current method has been found to be still sufficiently accurate.

Reference [10] presents two time domain techniques for load modeling in the presence of harmonics. In the first techniques, the harmonic contents of the load voltage and current waveforms are assumed to be known in advance. While, the second technique uses directly samples of load voltage and current waveforms. The least error squares parameter estimation algorithm is used to estimate the load resistance, inductance and capacitance. Residual current, results from incomplete extraction of the parameters for all significant signal frequencies, is also estimated. Harmonics that are not common between load voltage and current waveforms are also identified.

Effects of load modeling for undervoltage load shedding are presented in Reference [11]. A dynamic and static load models are presented for showing how much load be to shed and what influence different load models have on the simulation result as well as on the analysis.

The analysis of the load harmonic model under unbalanced condition is presented in Reference [13]. Different load models at the frequency domain have been derived for linear and nonlinear loads. The models of linear and non-linear loads are incorporated to the three-phase harmonic load flow formulation.

An approach to non-linear load modeling in the time domain has been developed in Reference [14]. The nonlinear load model accommodates both steady state and dynamic behavior. Two models are used; the first one is a polynomial function of second order of normalized voltage for the active and reactive powers, while in the second model the active and reactive powers are nonlinear functions of the normalized voltage and the frequency deviation. The

proposed load models are capable of representing the interaction between network conditions and load.

Reference [15] presents the use of chaotic dynamics to describe the operation of nonlinear loads. The proposed approach is applied to electric arc furnace. A tuning mode is described to develop the parameters of a chaotic model, this model is trained to have time and frequency response that are tuned match the current from the arc furnace.

Reference [16] presents a fast and efficient on-line identification of non-linear loads in power systems. The proposed technique is an application of a discrete time-dynamic filter based on stochastic estimation theory which is suitable for estimating parameters on-line. The algorithm uses sets of digital samples of the distorted voltage and current waveforms of the non-linear load to estimate the harmonic contents of these two signals. The non-linear load admittance is then calculated from these contents.

This paper presents an approach based on the Z-transform to model single phase electric loads in the presence of harmonics. The impedance or admittance of the load is expressed as linear autoregressive exogenous input (ARX). The parameters of this series are identified using the least error squares (LES) estimation algorithm, where the digitized samples of the load input voltage and the resulting output current are used. Having identified the ARX parameters model, the impedance, or admittance of the equivalent circuit of the load can be expressed in time domain. Using the well-known definition of the perunit Z, the admittance of the load can be written in the frequency domain. The proposed algorithm can easily be used to model the impedance or admittance of the electric distribution network on the harmonic polluted environment, as well as the transmission network, for load flow, stability analysis and harmonics filter design. Results of simulated and actual recorded data for modeling single-phase loads in the presence of harmonics are reported in this paper

II. MODELING SINGLE PHASE LOADS

The current is written as ARX series with the previous current samples are taken into consideration. The input voltage applied and the resulting current produced can be written as an ARX series as [18]

$$i(t) + a_1 i(t - 1) + a_2 i(t - 2) + \dots + a_m i(t - m) = b_0 v(t) + b_1 v(t - 1) + b_2 v(t - 2) + \dots + b_n v(t - n) \quad (1)$$

Where a_1, a_2, a_m and b_1, b_2, \dots, b_m are the load parameter to be estimated. Using the unit operator Z defined as: $Z^{-1}v(t) = v(t-1)$ and $Z^{-m}v(t) = v(t-m)$, then equation(6) can be written as

$$(1 + a_1 Z^{-1} + a_2 Z^{-2} + a_3 Z^{-3} + \dots + a_m Z^{-m}) i(z) = (b_0 + b_1 Z^{-1} + b_2 Z^{-2} + \dots + b_n Z^{-n}) v(z) \quad (2)$$

The load admittance, which can be considered as a transfer function, can be written as:

$$Y(z) = \frac{i(z)}{v(z)} = \frac{N(Z)}{D(Z)} \quad (3)$$

Where

$$N(Z) = (b_0 + b_1 Z^{-1} + b_2 Z^{-2} + \dots + b_n Z^{-n}) \quad (4)$$

And

$$D(Z) = (1 + a_1 Z^{-1} + a_2 Z^{-2} + a_3 Z^{-3} + \dots + a_m Z^{-m}) \quad (5)$$

Equation (1) can be written in a vector form as:

$$\underline{I}(\tau) = A(\tau) \underline{X} + \underline{g}(\tau) \quad (6)$$

Where

$\underline{I}(t)$ is an $m \times 1$ vector samples of the current

$A(t)$ is the $m \times (m+n)$ matrix of measurements whose elements contain the previous and recent current and voltage samples

\underline{X} is the $(m+n) \times 1$ load model parameters to be identified and

$\underline{g}(\tau)$ is $m \times 1$ errors vector to be minimized.

The solution to equation (11) based on least error squares algorithm is

$$\underline{X} = [A^T(t) A(t)]^{-1} A^T(t) \underline{I}(t) \quad (7)$$

Having identified the load model parameters from equation (7), then the load admittance of equation (3) can be

calculated at any harmonic frequency f , just by replacing Z by $Z = \exp(-j2\pi \frac{f}{f_s})$, with f_s is the sampling

frequency used to obtain the earlier required samples in equation (1).

Having identified the parameters vector $\underline{X}(t)$, then the admittance $Y(\omega)$ of the load model can be calculated from equations (3) to (5), by substituting for the value of Z in these equations. Note that after substitution for Z ,

these equations turn out to be complex equations. The value of $Y(\omega)$ is the amplitude of $\frac{N(\omega)}{D(\omega)}$, and the phase

angle is the difference between the angles of $N(\omega)$ and $D(\omega)$.

III. RESULTS FOR MODEL [10]

The test example is an actual recorded data for a large 1250 HP induction motor connected to a 44 kV power system bus [10]. A 6-pulse converter type drives the motor, which is the source of harmonics. The load voltage and current waveforms are given in Figure (1). The harmonics contents of these waveforms are:

$$v(t) = 0.2822 \cos(\omega t - 28.6^\circ) + 0.0016 \cos(3\omega t - 22.5^\circ) + 0.01152 \cos(5\omega t - 162.4^\circ) + 0.01274 \cos(7\omega t + 145.2^\circ)$$

$$i(t) = -\cos(\omega t + 100^\circ) + 0.027 \cos(3\omega t) + 0.151 \cos(5\omega t) + 0.1335 \cos(7\omega t)$$

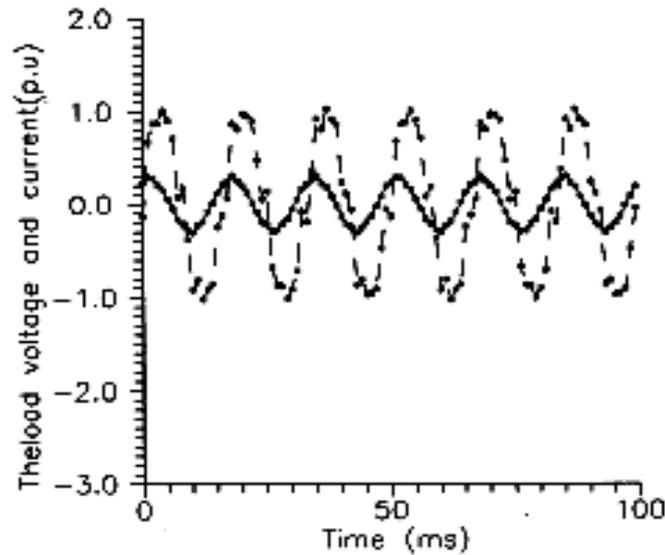


Figure 1 Voltage and current waveforms

The model of the admittance, described by equation (3), is used in this section to model the harmonic load described by the voltage and current waveforms. The order of the numerator is chosen to be fourth order, while the denominator is chosen to be fifth order in Z. The parameters of the admittance transfer function are estimated to be:

$$b_0=0.3.1515, b_1= 1.388, b_2 = -0.9424, b_3=0.9365, b_4 = -0.4314, a_1= -0.445, a_2 = -0.749, a_3 = 0.00805, a_4 = -0.1094, a_5 = -0.0435.$$

Figure (2) gives the variation of the load conductance with the harmonics frequency. The solid curve gives the polynomial fit to this conductance. A 10-order polynomial is used, which has the function:

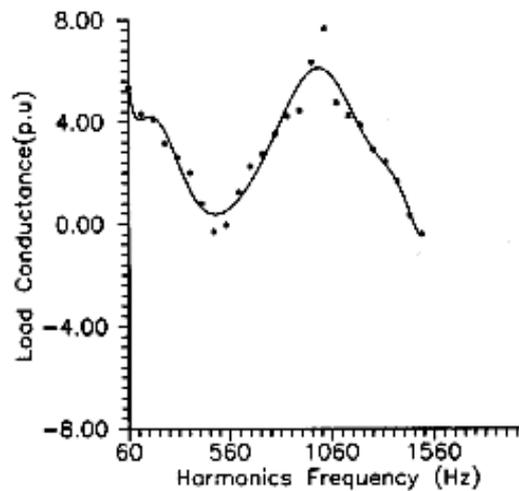


Figure 2 Variation of load conductance with frequency

*** Estimated _____ best fit

$$G(f)= 16.02 -0.344 f +0.0038 f^2 - 2.26 \times 10^{-5} f^3 + 7.48 \times 10^{-8} f^4 -1.53 \times 10^{-10} f^5 + 2.02 \times 10^{-13} f^6 - 1.704 \times 10^{-16} f^7 + 8.92 \times 10^{-20} f^8 -2.632 \times 10^{-23} f^9 + 3.341 \times 10^{-27} f^{10}$$

Figure (3) gives the load susceptance variation with harmonic frequencies. The asterisks in this figure give the estimated conductance, while the solid curve gives the best fit to the curve. We use a polynomial of order 10 to obtain this best fit, which is gives as:

$$B(f)=1.56 + 0.059f - 0.00115 f^2 + 9.06 \times 10^{-6} f^3 -3.68 \times 10^{-8} f^4 + 8.63 \times 10^{-11} f^5 - 1.248 \times 10^{-13} f^6 +1.134 \times 10^{-16} f^7 - 6.313 \times 10^{-20} f^8 + 1.969 \times 10^{-23} f^9 - 2.263 \times 10^{-27} f^{10}.$$

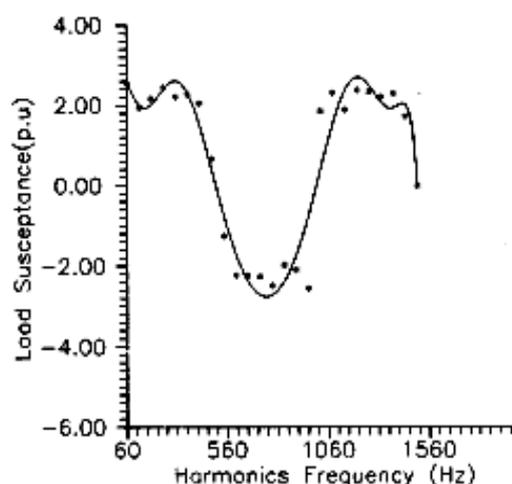


Figure 3 Variation of load susceptance with frequency
*** Estimated _____ best fit

Examining the two figures and the fitted equations for the conductance and the susceptance, one can notice that:

The load has a dc conductance as well as an a.c conductance that depends on the harmonic frequency.

- The load conductance is always positive irrespective to the frequency in question.
- The load susceptance changes its sign depending on the harmonics frequency.

The load has many resonance frequencies, where the value of the susceptance equals zero.

IV. CONCLUSIONS

The harmonic model of a single-phase load is presented in this paper, in the time domain. The model is a rational function of the load admittance, where the recent and previous samples of the voltage and current waveforms are used. The least errors square algorithm is used to estimate the parameters of the load. A set of polynomial equations for the conductance and the susceptance of the model are derived. It has been noticed that the proposed model is suitable for the harmonic analysis of the system that contains such nonlinear loads. The model can be used in harmonic power flow analysis, and system stability analysis.

The developed model in this paper is for single-phase loads. It is worthwhile to develop a suitable model for a three phase unbalanced load, which is our current research.

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